ANTENNA LABORATORY

Technical Report No. 59

THEORETICAL STUDY OF A CLASS OF LOGARITHMICALLY PERIODIC CIRCUITS

by
Raj Mittra

Contract No. AF33(657)-8460
Project No. 6278, Task No. 40572

JULY 1962

Sponsored by
AERONAUTICAL SYSTEMS DIVISION
WRIGHT-PATTERSON AIR FORCE BASE, OHIO

ELECTRICAL ENGINEERING RESEARCH LABORATORY
ENGINEERING EXPERIMENT STATION
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CONTENTS

ILLUSTRATIONS

1. INTRODUCTION

Many log-periodic distributed circuits in the form of various antenna structures have been investigated experimentally be DuHamel[1], Isbell[2], Carrel[3], Mayes[4], and others. Theoretical investigations of LP circuits have been made by DuHamel, Carrel, and Mayes, et. al. DuHamel has analyzed a particular lumped type of LP circuit composed of an infinite number of admittances in parallel and has given closed form expressions for the representation of the admittance of such circuits. In a later work he has also studied some general properties of LP structures. Carrel has done a considerable amount of numerical work in connection with the LP antenna structure with a finite number of elements.

The purpose of this paper is threefold:

a) to derive closed form expressions for lumped Foster type LP circuits.
b) to derive the characteristic equation for an infinite log-periodically loaded transmission line and to discuss a method of solution of the above equation.
c) to present a general approach towards studying a class of LP structures in terms of the Brillouin (k-β) diagrams of the corresponding simply periodic structures.

2. LUMPED LOG PERIODIC CIRCUITS OF FOSTER TYPE

In the first part of this paper we shall derive a closed form expression for the lumped Foster type LP network shown in Figure 1.

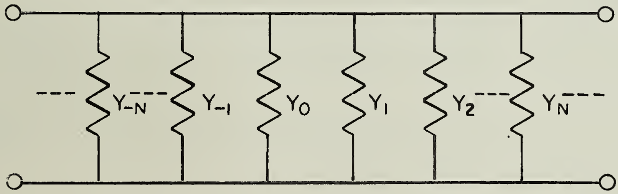

Figure 1. First order Foster type LP network

We shall consider the first order circuit first and will subsequently extend our analysis to the r^{th} order network. A first order LP Foster type of circuit will be defined as one which has the representation

$$Y_{in}(S) = \sum_{n=-\infty}^{\infty} \frac{1}{R + \alpha^n SL + \dfrac{1}{\alpha^n SC}} \tag{1}$$

where R, L and C are constants and $S = \sigma + jw$, or the complex frequency.

It is easily verified that Y_{in} is log-periodic, i.e.,

$$Y_{in}(\alpha S) = Y_{in}(S) \tag{2}$$

and α is identified as the log period of the network. An r^{th} order circuit of the same type admits a general representation

$$Y_{in}^{(n)}(S) = \sum_{q=1}^{r} \sum_{n=-\infty}^{\infty} \frac{A_q}{R_q + \alpha^n SL_q + \dfrac{1}{\alpha^n SC_q}} \tag{3}$$

As a first step toward deriving a closed form expression for the first order network we shall prove a Lemma, followed by a theorem applicable to such structures.

Lemma:

The driving point admittance function of a Foster type log-periodic admittance network is pure real for two infinite sequence of frequencies $S = \pm jw_o a^n$ and $S = \pm jw_o a^{n+1/2}$ where w_o is a constant depending on the network and n goes from $-\infty$ to $+\infty$ through integral values.

Proof:

From Equation (1) we obtain by substituting $S = jw$

$$Y_{in}(jw) = \sum_{n=-\infty}^{\infty} \frac{1}{R + j\left(a^n wL - \frac{1}{a^n w_C}\right)} = \frac{1}{w_o L} \sum_{n=-\infty}^{\infty} \frac{1}{D_o + j\left(a^n \frac{w}{w_o} - \frac{w_o}{a^n w}\right)} \tag{4}$$

$$= g_{in} + jb_{in}$$

where

$$w_o = \frac{1}{\sqrt{jLc}} \quad , \quad D_o = \frac{R}{w_o L}$$

Y_{in} may be re-written as

$$w_o L\, Y_{in} = g_{in} + jb_{in} = \sum_{n=-\infty}^{\infty} \frac{D_o}{D_o^{\,2} + \left(a^n \frac{w}{w_o} - \frac{w_o}{w\, a^n}\right)^2} - j \sum_{n=-\infty}^{\infty} \frac{a^n \frac{w}{w_o} - \frac{w_o}{w\, a^n}}{D_o^{\,2} + \left(a^n \frac{w}{w_o} - \frac{w_o}{w\, a^n}\right)^2} \tag{5}$$

Now we shall show that

$$b_{in} = 0 \quad \text{for} \quad \omega = a^n w_o \,, \qquad \text{n goes from } -\infty \text{ to } \infty$$

$$\text{and for} \quad w = a^{n+1/2}\, w_o$$

First by substituting $w = w_o$ in Equation (3) we get

$$b_{in}\Big|_{w=w_o} = -\sum_{n=-\infty}^{\infty} \frac{a^n - a^{-n}}{D_o^2 + \left(a^n - a^{-n}\right)^2} \tag{6}$$

It is easily verified that $b_{in}\big|_{w=w_o} = 0$ since the $n = m$ term in the sum cancels with the $n = -m$ term, and the $n = 0$ term is identically zero.

From the log-periodic nature of Y_{in} it is obvious that $Y_{in} = 0$ for

$$w = a^n w_o \qquad \text{where n goes from } -\infty \text{ to } \infty$$

Next put $w = w_o a^{1/2}$ in Equation (5) obtaining

$$b_{in}\Big|_{w=a^{1/2} w_o} = -j \sum_{n=-\infty}^{\infty} \left(a^{n+\frac{1}{2}} - a^{-(n+\frac{1}{2})}\right) \Big/ \left\{ D_o^2 + \left(a^{n+\frac{1}{2}} - a^{-(n+\frac{1}{2})}\right)^2 \right\} \tag{7}$$

Again it is verified that $b_{in} = 0$ since the term corresponding to $n = m$ in the sum cancels with the $n = -(m+1)$ term.

It is a trivial step to extend the result to the negative frequencies by using the relation

$$b_{in}(w) = -b_{in}(-w)$$

Hence the Lemma is proved.

Theorem 1:

A general first order log-periodic circuit of Foster type is equivalent at its input terminals to a pure resistance network terminated by a purely lossless two terminal log-periodic network with a driving point reactance function of Foster type.

Proof:

From the form of $Y_{in}(S)$ given in Equation (1) we see that Y_{in} has denumerably infinite numbers of poles and zeros in the complex S-plane. Also $Y_{in}(S)$ is analytic on and in the neighborhood of the imaginary axis, $S = jw$. Let

$Y'_{in}(jw)$ be a function which matches $Y_{in}(jw)$ at denumerably infinite number of points, i.e.

$$Y_{in}(jw_r) = Y'_{in}(jw_r) \qquad \text{for } r = 1,\ 2,\ \ldots\ \infty$$
$$= 1,\ -2,\ \ldots\ -\infty$$

Where w_r's may be located arbitrarily. Then from the well known sampling theorem which may be extended for the case of non-uniform sampling, one may prove

$$Y_{in}(jw) = Y'_{in}(jw)$$

Since $Y_{in}(jw)$ is identical to $Y'_{in}(jw)$ for all w through analytic continuation we may show that

$$Y_{in}(S) = Y_{in}(S)$$

Next we show that we can construct a Y_{in} which matches Y_{in} for a denumerably infinite number of points.

We have already shown that Y_{in} is pure real for $S = \pm j\alpha^n w_o$ and $S = \pm j\alpha^{n+\frac{1}{2}} w_o$.

$$\left.\begin{aligned} Y_{in} &= g_1 \ \text{ for } S = \pm j\alpha^n w_o \\ &= g_1 + g_2 \ \text{ for } S = \pm j\alpha^{n+\frac{1}{2}} w_o \end{aligned}\right\} \quad n \text{ goes from } -\infty \text{ to } \infty \qquad (8)$$

Now let Y'_{in} be the driving point impedance of a network structure shown in Figure 2.

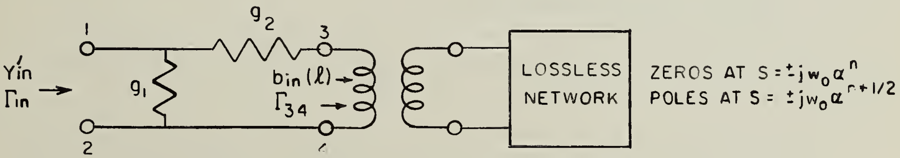

Figure 2. Equivalent circuit for a general first order Foster type LP network

Then a little deliberation will show that

$$Y_{in}(jw) = Y'_{in}(jw) \quad \left.\begin{array}{l} \text{for } w = \pm\, \alpha^{n+\frac{1}{2}}\, w_o \\ \text{and } w = \pm\, \alpha^{n}\, w_o \end{array}\right\} \text{ n goes from } -\infty \text{ to } \infty \qquad (9)$$

and that the lossless LP network characterized by zeros and poles at $S = \pm\, jw_o\alpha^{n+1/2}$ and $S = \pm\, jw_o\alpha^n$, respectively is a physically realizable Foster type log-periodic network. Hence the theorem is proved. It should be noted that the turnsratio of the lossless transformer has to be determined through one additional piece of information.

As a next step we derive a closed form for $b_{in}^{(1)}$ which is the input susceptance of the terminating lossless network described above. Its pole and zero locations are given by:

$$\begin{array}{ll} \text{zeros at } w = \pm\, w_o\alpha^n & \\ \text{and} & n = -\infty, \ldots -2, -1, 0, 1, 2, \ldots \infty \\ \text{poles at } w = \pm\, \alpha^{n+\frac{1}{2}} & \end{array} \qquad (10)$$

Because of the nature of the location of the poles and zeros, it will be found convenient to consider a variable W defined by

$$W = \ln w/w_o \qquad (11)$$

Then the location of poles and zeros, rewritten in terms of the new variable, become doubly periodic as seen from the following:

$$\text{zeros at } W = n\Gamma + jm\pi \qquad \begin{array}{l} n = -\infty, -2, -1, 0, 1, 2 \ldots \infty \\ m = \text{arbitrary integer or zero} \end{array} \qquad (12a)$$

$$\text{poles at } W = (n + \tfrac{1}{2})\Gamma + jm\pi \qquad \text{n and m having the same range as in (12a)} \qquad (12b)$$

where $\Gamma = \ln \alpha$.

Functions exhibiting such double periodicity may be expressed in terms of the Elliptic Functions. DuHamel was first to recognize this property of the

LP Foster type networks. Now consider the infinite product representation for the elliptic function sn(u, k) given by

$$\mathrm{sn}(u,k) = \frac{2K}{\pi} \sin (\pi\, u/2\, K) \prod_{n=1}^{\infty} \left[\frac{1-q^{2n-1}}{1-q^{2n}}\right]^2 \left[\frac{1 - 2q^{2n} \cos (\pi\, u/K) + q^{4n}}{1 - 2q^{2n-1} \cos(\pi\, u/K) + q^{4n-2}}\right] \qquad (13)$$

$q = e^{-\pi/2\, K'/K}$, $K = \mathrm{sn}^{-1}(1,k)$, $K' = \mathrm{sn}^{-1}(1,k')$, $k' = \sqrt{1-k^2}$

It is seen that the function has zeros at

$$-j\,\frac{\pi u}{2K} = jm\pi \pm n \ln q \qquad n = 0, 1, 2, \ldots \infty, \; m = 0, \pm 1, \pm 2 \qquad (14a)$$

and the poles are located at

$$\frac{-j\pi u}{2K} = jm\pi \pm (n + \tfrac{1}{2}) \ln q \qquad n = 0, 1, \ldots \infty, \; m = 0, \pm 1, \pm 2 \ldots \qquad (14b)$$

Hence we may construct a representation for $b_{in}(1)$ as follows:

$$b_{in}(1) = C\, \mathrm{sn}\left(\frac{j\, 2K\, w}{\pi}, k\right) \qquad (15)$$

$$= C\, \mathrm{sn}\left(\frac{j\, 2K \ln w/w_o,\; k}{\pi}\right)$$

where C is a constant as yet undetermined. It may be determined if $b_{in}(1)$ is known for an additional value of w. However, the constant C may be conveniently absorbed in the transformer turns ratio r (see Figure 2). Equation (15) is of the same form as obtained by DuHamel for a first order lossless Foster LP network.

It has now been shown that the general first order LP network may be represented by a structure shown in Figure 2. We are now ready to state another theorem concerning the general first order LP Foster network.

Theorem 2:

The locus of the input reflection coefficient Γ_{in} of a first order LP network as a function of frequency is a circle.

Proof:

The proof is based on the equivalent representation of the LP first order Foster network, shown in Figure 2. Since the locus of Γ_{in} as seen at the pair of terminals 3-4 is obviously a circle (unit circle) one immediately observes that the plot of Γ_{in} is measured at the terminals 1-2 must also be a circle because of the property of bilinear transformation. This proves the theorem.

An expression for Y_{in} for a general LP structure of first order may also be derived from the equivalent representation. The expression is

$$Y_{in} = g_1 + \frac{jg_2\, r^2\, \mathrm{sn}\left(\frac{j2K}{\pi} \ln w/w_o,\ k\right)}{g_2 + j\, r^2\, \mathrm{sn}\left(\frac{j2K}{\pi} \ln w/w_o,\ k\right)} \tag{16}$$

The constant r is obtainable through the calculation of Y_{in} for any single convenient value of w.

We shall conclude this section with a brief comment on the r^{th} order LP network of Foster type. By combining two or more first order networks in parallel, a r^{th} order network is constructed. It is easily shown that the input reflection coefficient is not necessarily a circle. A typical plot of the input reflection coefficient for a second order network is shown in Figure 3.

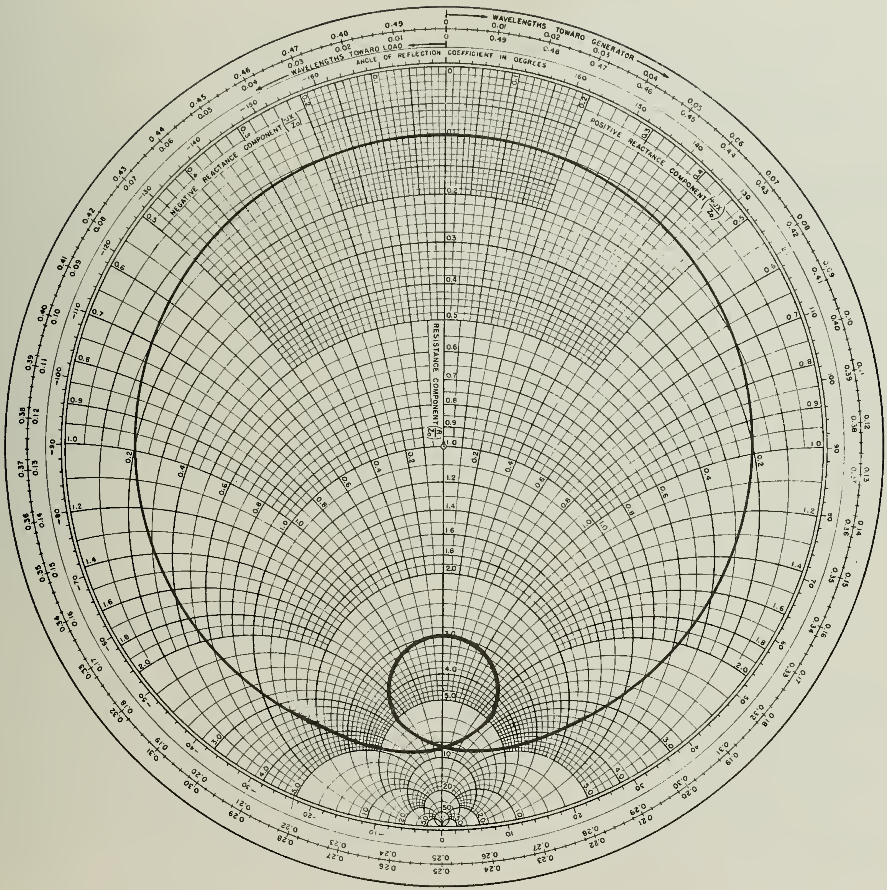

Figure 3. Typical plot for Γ_{in} for a second order Foster LP network

3. LOG PERIODICALLY LOADED TRANSMISSION LINE

We shall now analyze the log-periodically loaded transmission line structure. It is hoped that this will lead to an understanding of the behavior of the LP dipole antenna by seeking extensions of the basic ideas developed here.

Consider a LP loaded transmission line composed of lumped and distributed circuits of the form shown in Figure 4.

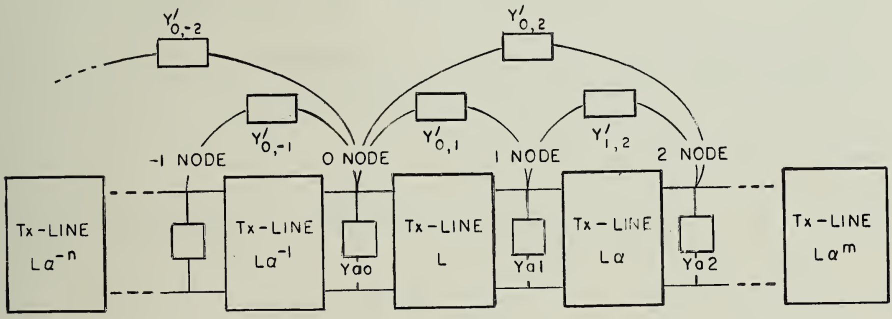

Figure 4. LP loaded transmission line

Note that the line length is scaled by a factor a in each cell and thus the line length tends to zero for large n in the negative direction from the zeroeth cell. Similarly the line length increases in the possible direction as is to be expected. In addition, the loading admittances Y_{an}'s and the mutual admittances $Y_{p,q}$ must satisfy the log-periodic property. This implies that

$$Y_{an} = Y_{a,n+r} \; (k\, a^{r}) \tag{17a}$$

$$Y'_{n,n+q}(k) = Y_{p,p+q} \; (k\, a^{p-n}) \tag{17b}$$

$k = 2\pi/\lambda$ = wave number. We shall now derive the characteristic equation for the LP loaded line. To this end consider the 0^{th} node and derive the equation

$$0 = \ldots + Y_{-2,0} V_{-2} + Y_{-1,0} V_{-1} + Y_{0,0} V_0 + Y_{0,1} V_1 + Y_{0,2} V_2 + \ldots \tag{18}$$

where

$$Y_{00} = Y_{a0} + \sum_{m=-\infty}^{\infty} Y'_{0,m} - j(\cot kL + \cot kL\, \alpha^{-1})$$

$$Y_{0,r} = j\,(\operatorname{cosec} kL\, \alpha^{-1}\, \delta_r^{-1} + \operatorname{cosec} kL\, \delta_r^{1}) + Y'_{0,r}$$

$\delta_r^{\,q} = 1$ for $q = r$ and zero otherwise. If the structure is uniform periodic, $\alpha = 1$ and if the mutual admittances $Y'_{n,r}$ are zero, Equation (15) reduces to

$$0 = j \operatorname{cosec} kL\, V_{-1} + (Y_a - 2j \cot kL)\, V_o + j \operatorname{cosec} kL\, V_1 \tag{19}$$

If β is defined as the propagation constant, we have

$$e^{-j\beta L} = \frac{V_{n+1}}{V_n}$$

for any n and Equation (19) may be rewritten as

$$0 = j \operatorname{cosec} kL\, e^{j\beta L} + (Y_a - 2j \cot kL) + j \operatorname{cosec} kL\, e^{-j\beta L} \tag{20}$$

If the loading admittance Y_a is a pure susceptance i.e. $Y_a = jb$, the following equation may be derived for β:

$$\cos \beta L = \cos kL - \frac{b \sin kL}{2} \tag{21}$$

Equation (21) is the well known characteristic equation for a uniformly periodic loaded line with no mutual coupling admittances $Y'_{p,q}$ between the nodes.

Coming back to the LP structure, we define

$$\frac{V_1}{V_0} = \gamma(k) \tag{22}$$

Then using the LP properties of the structure it is readily shown that

$$\frac{V_{p+1}}{V_p} = \gamma\ (k\alpha^p) \tag{23}$$

Equation (23) has the following implication. If $\gamma(k)$ is known at all log-periodic frequencies $k_o\alpha^q$ for all positive and negative q, then the ratio of any two adjacent cell voltages are obtainable from this information for any given $k = k_o$. If $\gamma(k)$ is known for all k, then of course, we know the behavior of the structure at all points at all frequencies. In the following we shall detail a procedure for calculating $\gamma(k)$ in terms of the circuit parameters of the structure under consideration.

Use Equations (22), (23), and rewrite Equation (18) as

$$\begin{aligned} \ldots + Y_{-2,0}\ \gamma^{-1}(k\alpha^{-2})\ \gamma^{-1}(k\alpha^{-1}) + Y_{0,-1}\ \gamma^{-1}(k\alpha^{-1}) + Y_{0,0} + Y_{0,1}\gamma(k) + \\ Y_{0,2}\gamma(k)\ \gamma(k\alpha) + \ldots \end{aligned} \tag{24}$$

Equation (24) is the desired characteristic equation for $\gamma(k)$. A solution of the above equation will now be discussed. As a simplifying step assume that the mutual admittances $Y'_{n,m}$'s are negligibly small. Further, let the asymptotic behavior of the loading elements be such that they satisfy

$$Y_{a,t} \longrightarrow 0 \qquad \text{for t large}$$

for k finite. This has the implication that the elements away from the center region and to the right hand side of it have negligible loading effect although the same cannot be said in general about the loading aspect in the left hand side. This is because the line length $L\alpha^{-t}$ also goes to zero for a large t and the effect of Y_{at} may still be that of a uniform loading per unit length of the line even if $Y_{at} \longrightarrow 0$ for large negative t as well.

Neglecting the mutual terms Y'_{mn}, Equation (21) may be simplified to

$$0 = j\gamma^{-1}(k\alpha^{-1})\ \text{cosec}(kL\alpha^{-1}) + \left\{ Y_{a0} - j(\cos kL + \cot kL\alpha^{-1}) \right\} + j\gamma(k)\ \text{cosec}\ kL \tag{25}$$

which may be rewritten as

$$\gamma(k\alpha^{-1}) = \frac{-j \operatorname{cosec} kL\alpha^{-1}}{Y_{ao}(k) - j\,(\cot kL + \cot kL\alpha^{-1}) + j\gamma(k) \operatorname{cosec} kL} \tag{26}$$

for cell 0 and in general

$$\gamma(k\alpha^{-1+r}) = \frac{-j \operatorname{cosec}\,(kL\alpha^{-1+r})}{Y_{a,r}\,(k\alpha^{r}) - j\,(\cot kL\alpha\ + \operatorname{cosec} kL\alpha^{-1+r}) + j\gamma(k\alpha^{r}) \operatorname{cosec}\,(kL\alpha^{r})} \tag{27}$$

for cell 'r'. If r is large and positive $Y_{a,r} \longrightarrow 0$ under the assumption stated above. Under this condition Equation (27) asymptotically tends to

$$\gamma(k\alpha^{-1+r}) = \frac{-\operatorname{cosec}\,(kL\alpha^{-1+r})}{-\cot\,(kL\alpha^{r}) - \operatorname{cosec}(kL\alpha^{-1+r}) + \gamma\,(k\alpha^{r}) \operatorname{cosec}\,(kL\alpha^{r})} \tag{28}$$

which admits the solution

$$\gamma(k\alpha^{r}) = e^{-jkL\alpha^{r}} \tag{29}$$

for the outgoing wave. This may be easily verified by direct substitution. We may now use this known solution in Equation (28) and work backwards using this equation to calculate the various γ's in the form of the sequence

$$\gamma(k\alpha^{r-1}),\ \gamma(k\alpha^{r-2}),\ \ldots\ \gamma(k),\ \gamma(k\alpha^{-1}),\ \gamma(k\alpha^{-2}),\ \ldots\ \gamma(k\alpha^{-m}),\ \ldots$$

It is easily verifiable that the process will converge to a limit as the index m takes up increasing values. This is because the line length $L\alpha^{-m} \longrightarrow 0$ for a large m and $|\gamma(k\alpha^{-m})| \longrightarrow 1$. The phase also approaches the asymptotic behavior very rapidly and it is possible to extrapolate the curve for the phase of γ from the knowledge of its behavior for only moderate m's.

The alogrithm outlined here yields all the $\gamma(k\alpha^{r})$s and hence all the desired ratios of V_r/V_o. The current in the shunt element may also be obtained.

The problem of solving for the voltage and currents in a LP loaded transmission line of the type under consideration, may therefore be considered as solved.

Before we quote some numerical results based on the above procedure we would like to mention one other point which has to do with the effect of end reflection. To consider this effect, one starts with the solution

$$\gamma(k\alpha^r) = e^{jkL\alpha^r} \qquad \text{for large } r$$

instead of Equation (29). That Equation (30) is also a solution of Equation (28) may be easily verified. Numerical calculations show that the solution corresponding to this choice of asymptotic behavior in the right hand side does not affect very much the nature of the solution in the left hand side of the so called active region (region of high shunt current). The above is true for a reasonably efficient circuit i.e. one in which the current decays reasonably well beyond the active region. In effect therefore we may conclude that the end termination does not significantly affect the performance of an efficient LP structure.

Numerical results will be presented below in terms of graphs for voltage and current, etc. for a log-periodically loaded transmission line. The calculations are based on the Equation (28) developed above. The circuit parameters chosen are:

$$kL = 30^{\circ}$$

$$Y_{ao} = 1 \Big/ \left\{ 0.3 + j \left(w - \frac{1}{w} \right) \right\}$$

$$\alpha = 4$$

and it is assumed that the mutual coupling between the nodes is zero. The amplitude and phase distributions of the transmission line voltage and the shunt current are shown in Figures 5 and 6.

The general behavior of the solution for such circuits may be described as follows. Both the voltage and current decay rapidly beyond a certain point along the line. The element current becomes small also in the left

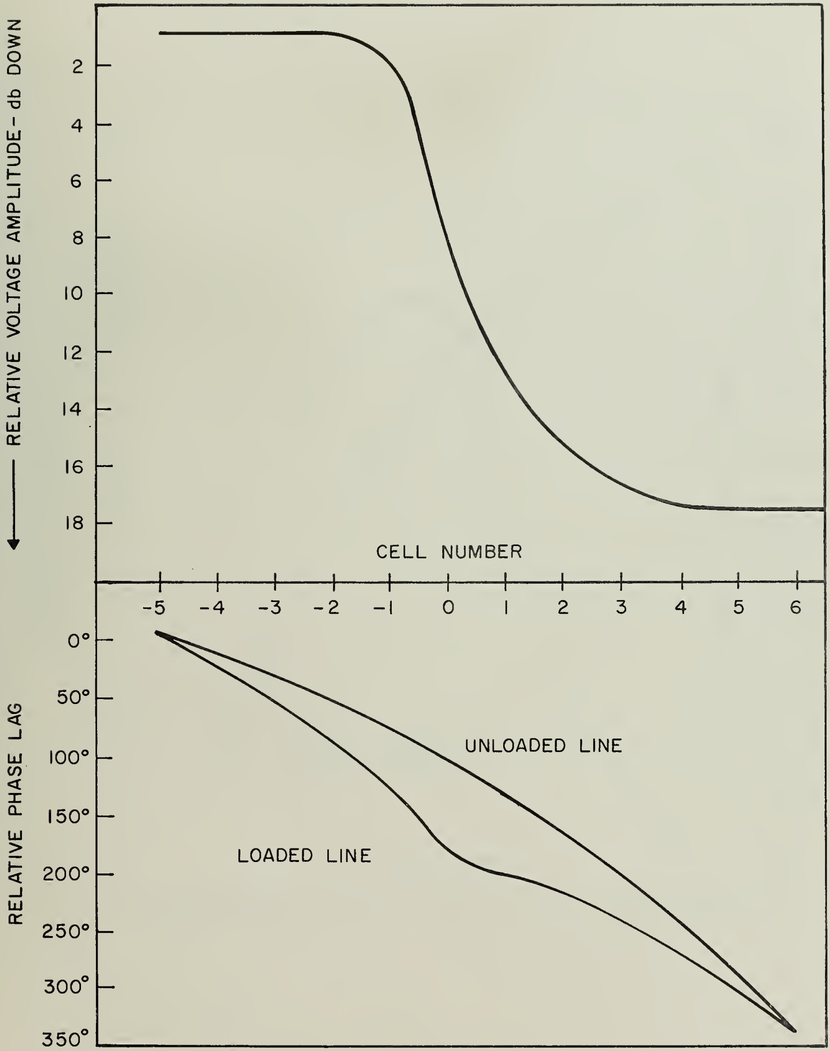

Figure 5. Cell voltage amplitude and phase plots for a loaded LP line

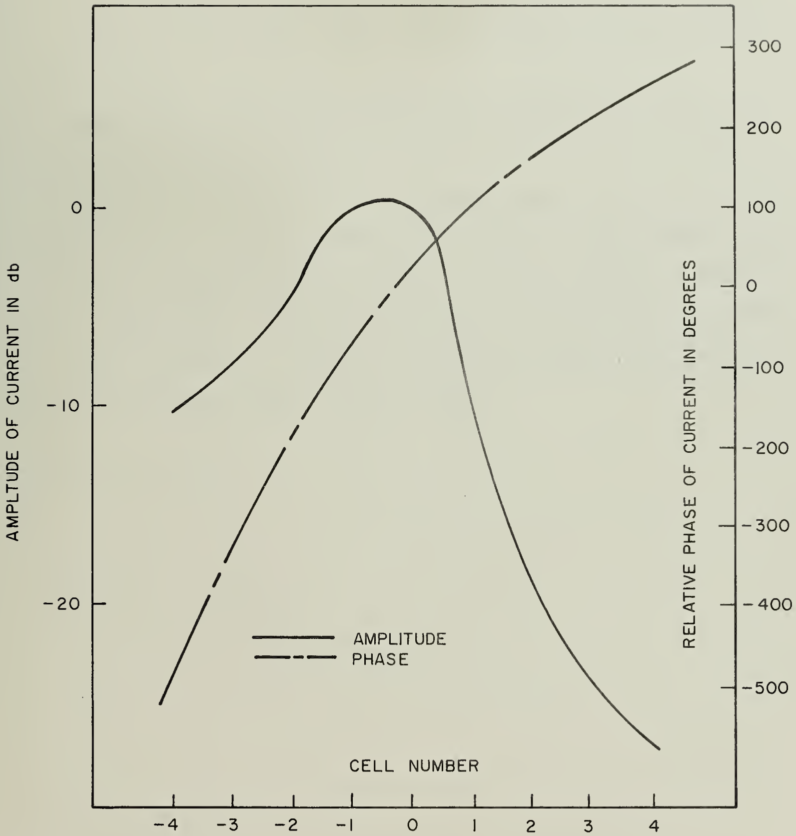

Figure 6. Amplitude and phase plots for a loaded LP line

hand side and has a peak in the middle, which is usually termed the active region. The voltage tneds to a constant towards the left hand end.

The phase curve shows that there exists a slow wave in the left hand side of the active region and that the effective phase constant, defined by the phase change per unit length, goes through a maximum in the neighborhood of the active region.

Although the model chosen here is rather an oversimplified one, it should be noted that similar characteristics have been observed by Carrel and others on the LP dipole antenna.

We shall close this section with one further comment concerning the extension of the above work for a general case where the mutual admittances are not zero. With one mutual term only, i.e., if the coupling is neglected between all but the adjacent nodes, the above procedure needs only a slight modification. Beyond that one has to go through a slightly more complicated process to solve the problem. However, there is no analytical difficulty in doing this and the details will be left out here.

4. THE BRILLOUIN DIAGRAM AND ITS APPLICATION TO TAPERED STRUCTURES

The Brillouin diagram describes the relationship between the wave number k and the phase propagation constant per cell of a simply periodic structure. In many cases, a complete knowledge of the diagram in an appropriate range of k, may be useful in predicting the behavior of tapered or modulated periodic structures derived from it. To illustrate the point, let us consider the case of a sinusoidally modulated reactance surface. The geometry of the surface and its k-β diagram is shown in Figure 7.

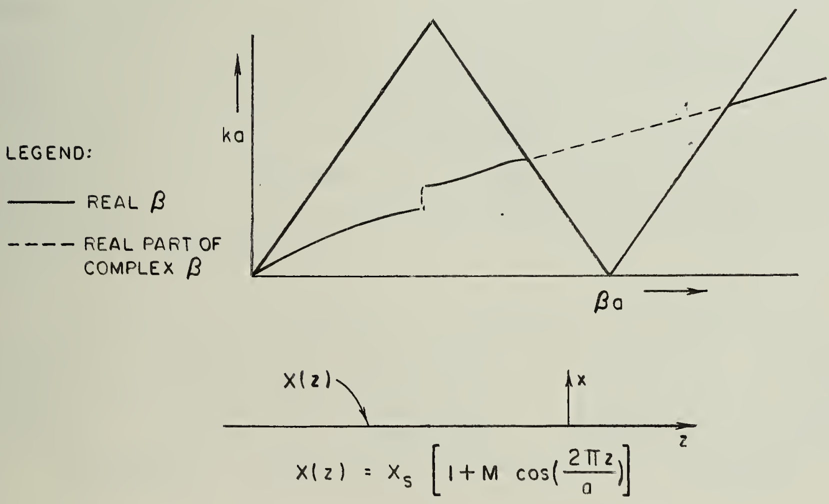

Figure 7. k-β diagram for a sinusoidally modulated reactance surface (after Oliner and Hessel)

Here X_s is the unmodulated surface reactance, M is the modulation index and a is the period of modulation. The ka-βa diagram for such a structure has been calculated in detail by Oliner and Hessel[5].

Now suppose the structure is tapered by gradually changing the period along the z-direction. The behavior of the tapered structure may still be predicted from the knowledge of the ka-βa diagram of the constant periodic structure, i.e., one with constant a. In the tapered structure, one expects bound surface wave fields for the region for which the local ka is small. As the local ka increases along the structure, there comes a region of attenuation corresponding to the vertical dashed line in the ka-βa diagram. Beyond this there is a propagation region again, which is followed by a leaky wave region corresponding to the dashed line outside of the first triangle of the k-β diagram. This general behavior repeats again for increasingly higher values of ka.

From the above discussion it is seen that a general estimate of the behavior of the tapered structure is possible in an approximate sense, in terms of the k-β diagram of its periodic counterparts.

To apply these ideas to the log periodically loaded transmission line, consider its periodic counterpart, viz. the simply periodic loaded line shown below in Figure 8. Y_a is the loading admittance and L is the length of the transmission line for each cell.

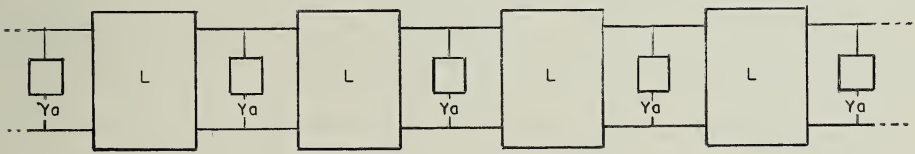

Figure 8. Simply periodic loaded transmission line

The characteristic equation for this structure was derived earlier in Equation (20), which may be written as

$$\cos \beta L = \cos kL + \frac{j}{2} Y_a \cdot \sin kL \qquad (30)$$

If Y_a is known a plot of kL versus βL may be readily made using Equation (30). Typical plots for various different types of Y_a are shown in Figures 9 and 10.

With lossless resonant circuits as shunt elements, the k-β diagram exhibits attenuation or stop bands, bounded by sharply defined values of kL outside of which the attenuation α is zero. The case of a lossless shunt loading circuit with two series resonances is shown by the dashed curve in Figure 9. The structure essentially behaves like a band stop filter. However, when loss is added to the loading element in the form of a series resistance, the corners of the lossless curve are rounded off as shown by the solid curves in Figure 9. There are no absolutely zero attenuation regions although high attenuation is still confined largely within bands centering the series resonant frequencies of the shunting load. With a single resonance type shunt load, the curve looks like the one shown in Figure 10, as of course is to be expected.

Actual experimental measurements made on a periodic monopole and dipole array made by Mayes* show a behavior very similar to the multiple resonance case described here. It is obvious, that, to obtain better correlation one should perform the calculations with a more exact representation of the dipole antenna rather than the simple one used here and should also include the effect of mutual admittances. Calculations along this line have been planned for the near future.

It is also of interest to compare the kL-βL diagram for dipole array calculated from the results reported by Carrel on LP structure. The diagram is shown in Figure 11. Local values of βL have been calculated from the amplitude and phase plot of the transmission line voltage for the LP array. It should be pointed out that because of considerably high attenuation in the active region of the LP array, Carrel's main region of interest was the first resonance region and our calculations end there. This corresponds to the single-mode type of operation of the array. The similarity between this diagram and Figure 10 is indeed striking when one considers the general shape of the curves.

It is possible to give a heuristic explanation of the performance of the LP structure in terms of the k-β diagram given in Figure 10. The structure supports slow waves (β>k) in the smaller kL regions and these are accompanied by little attenuation. With increasing kL, the band of high attenuation is reached. The phase βL reaches a turning point and subsequently reverses to

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* Private communication.

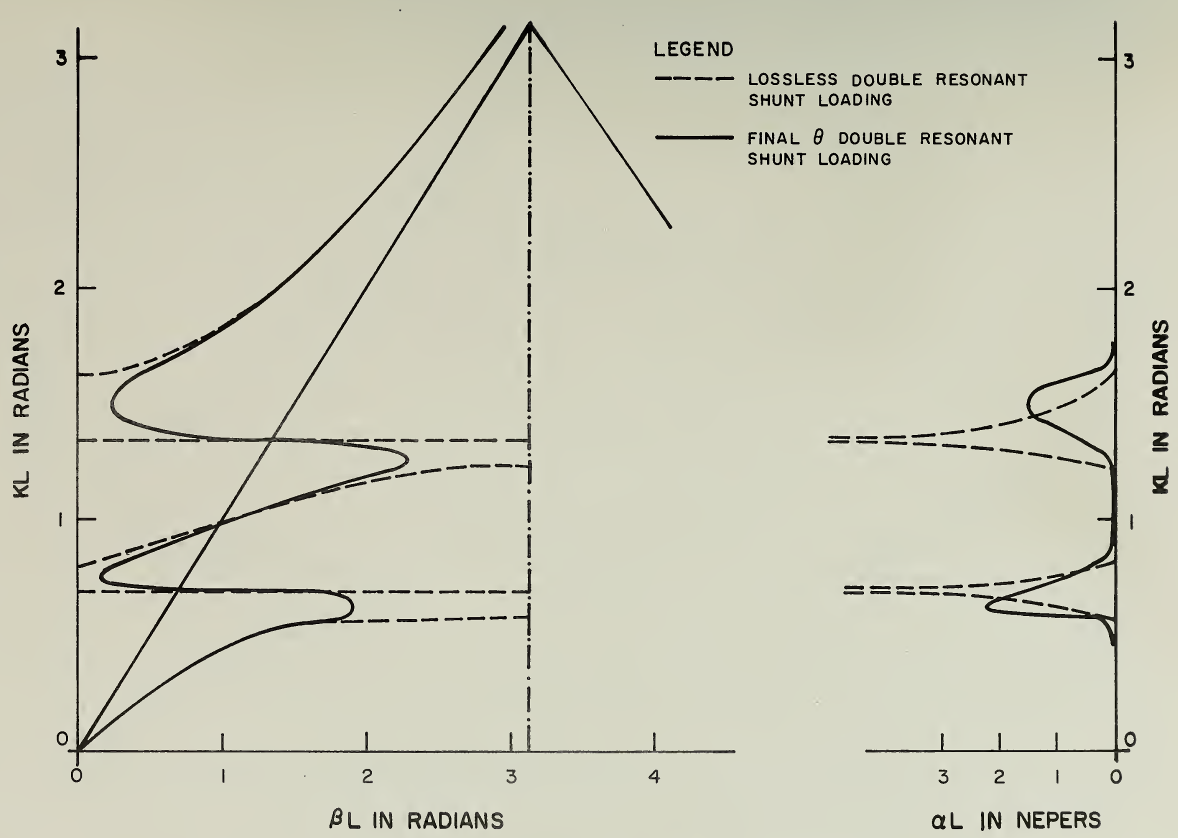

Figure 9. k-β diagram for a loaded transmission line

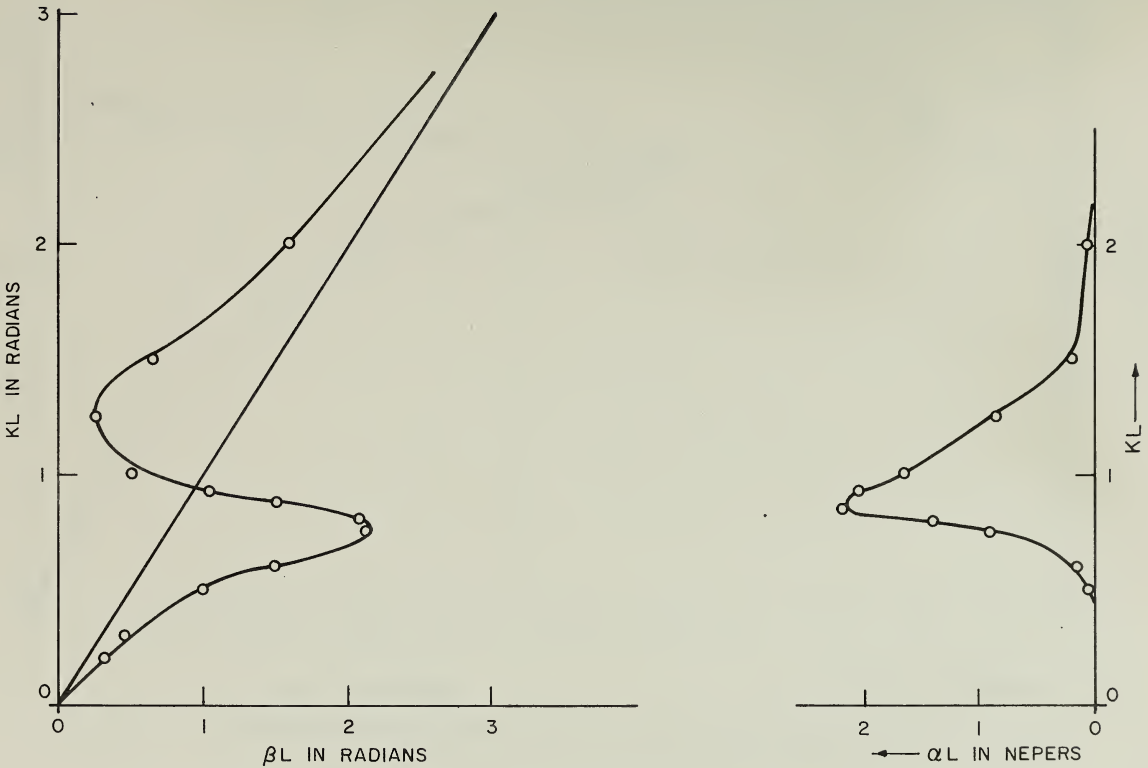

Figure 10. k-β characteristics for a lossy single resonant shunt loading

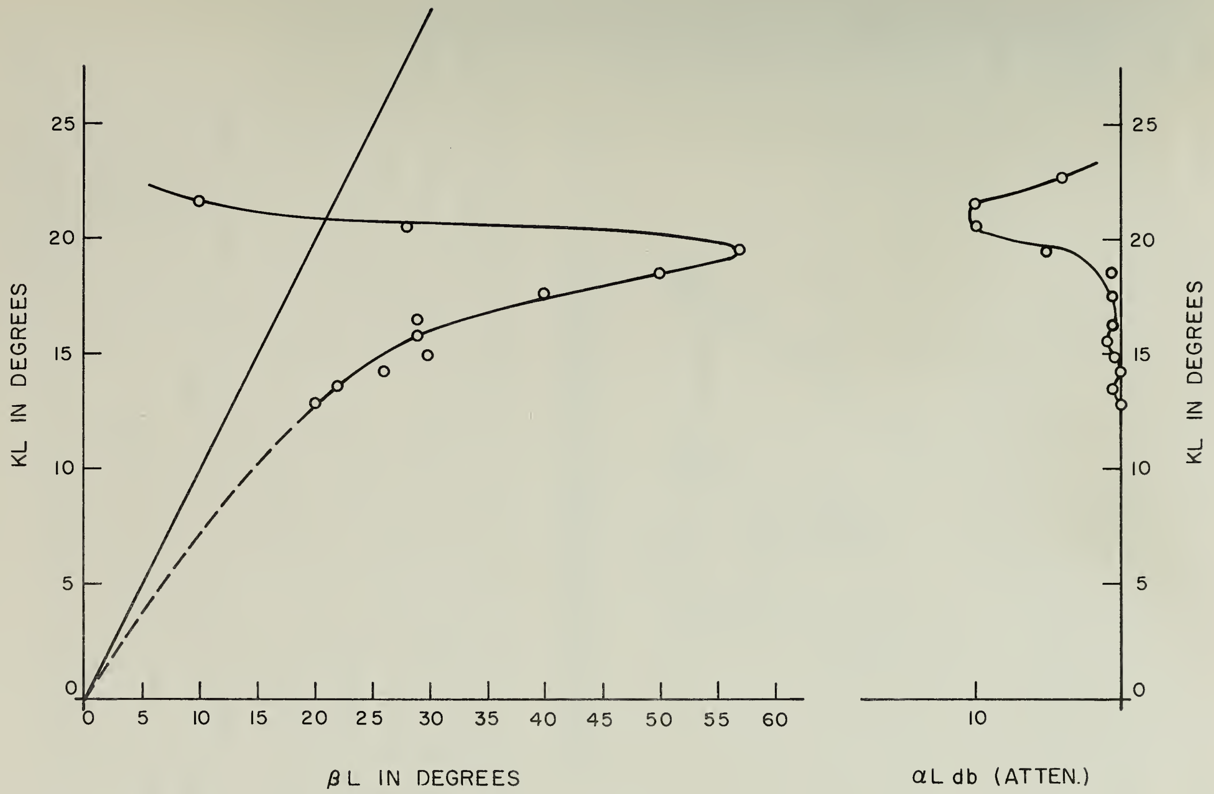

Figure 11. k-β diagram for a LP dipole antenna (calculations based on results by Carrel)

smaller values. All this is borne out by the calculations (see for instance, Figure 5) and measurements made on the LP structures of this type.

To explain the radiation in the backward direction by the LP dipole array which have 180° phase reversals in the alternate elements one should consider the phase plots of the currents rather than of the transmission line voltage. The current phase plot is readily obtained by shifting the transmission line voltage plot by 180°. This is shown in Figure 12.

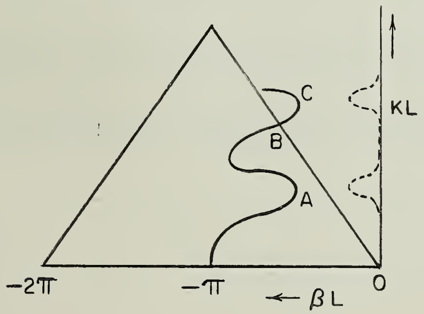

Figure 12. k-β diagram for the shunt current in a loaded line with additional 180° phase shift per cell

It is clear that near the attenuation regions A and C where the antenna is close to 1/2 wave and 3/2 wave resonance and is an effective radiator, the radiation pattern will favor the backward direction because of the nearness of these regions to the β=k line. In the neighborhood of the point B however, the antenna loading effect on the transmission line is negligibly small and hence energy is not efficiently coupled to the radiating structure from the feed structure. For the antenna this corresponds to 1-λ resonance region and not much radiation will occur in the neighborhood of crossing B.

It should be emphasized again that this is merely an attempt to explain the general picture of what happens in these structures and a more elaborate analysis is planned for the future on the actual dipole loaded transmission line. In an actual analysis the effect of the mutual coupling due to the criss-cross feed has to be taken into account but it is believed that this will not alter the general picture of the phenomenon which explains the behavior of these structures.

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